whether the SCIS model was run using the "average" or "incremental" set of cost assumptions. The impact of this assumption is important for several reasons:

(A) I have been involved in several analyses involving state commissions where it appeared that an LEC utilized the "average" version of one SCIS feature (or vertical service) module, while it used the "incremental" version in a different module for a similar service. By similar services I mean services that consume similar central office resources, e.g., line terminations or processor cycles. In some cases, the former "average" assumption was used for a feature associated with a monopoly service, whereas the latter type of SCIS run was performed for features that might be associated with competitive services such as "centrex" offerings.[2] Thus, the sensitivity of the SCIS output data to these types of assumptions must be part of the inquiry. Arthur Andersen's analysis should include the subsitution of "average" for "incremental" cost assumptions in SCIS runs to the extent this is feasible. If it is not feasible, Arthur Andersen should identify the reason(s) why, with respect to each BSE study item for each affected RBOC.[3]

(B) The term "incremental" can mean either that the SCIS study assumes the use of wholly new capacity by the vertical feature, or that the feature occupies otherwise spare capacity. The consequences of these different "incremental" capacity assumptions and/or average cost assumptions may be quite significant. The Bellcore presentation sheets used at the May 13 meeting illustrate the uses of "capacity" (see p."823.002"). Arthur Andersen must be able to report what assumptions, sets of conditions or other factors were applied to capacity costs by each RBOC with respect to each BSE study item. That is, the consultant's analysis should specify whether average, short-run incremental (i.e.,

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2. Many RBOCs now have tradenames associated with centrex, like Centron, Plexar or DCOSS.

3. To the extent that the audit does not or cannot identify this effect, the FCC should require each RBOC to file a *list* of each SCIS system basic or feature module that it has submitted as cost support with any state commission in the last three (3) years and state the assumptions, including "average" versus "incremental" that were used. This material would consist of lists, rather than the underlying SCIS cost runs and workpapers, that can be used for comparisons between rate setting techniques used for different RBOC central office features.

using spare) capacity or long-run incremental (i.e., using only added) capacity] was assumed for each study item.

(C) The same sheet from May 13 (p. 823.002) notes that the "fill factor" used to calculate usable capacity may be a time value calculation. Arthur Andersen should test the effects of different time values. Differences in RBOCs' discounting periods should, if possible, be equated to a common time horizon as well as a common discount rate. The Commission's Part 64 cost allocation rule requires that RBOC nonregulated services be assigned joint costs for the highest usage forecasted over a three-year period. The three year period is a relatively short-run horizon, but it is presumptively reasonable since the FCC has examined the issue in the context of joint cost accounting in CC Docket 86-111.[4] The time value fill factor assumptions used by the RBOCs should be rerun for each BSE study item using the three-year value.

Third, the November 26, 1991 petitions concerning the ONA tariffs identify vast differences in rate levels for BSEs whose estimated demand per RBOC is generally proportional to underlying BSA demand for the same carrier. "Call billing number delivery" is one good example. Thus, the analysis must consider the mix of facilities that each RBOC assumed would be used to serve the demand for each BSE. The Arthur Andersen study should identify the number of facility units (e.g. central office equipment) assumed by each RBOC for each BSE study item. The Bellcore portion of the May 13 presentation (p. 823.005) might be interpreted to suggest that any given BSE will have a facility usage value that is "hardwired" in the program (in this case, milliseconds). This is not the case, however, since facility parameters are input into several types of SCIS modules. Therefore Arthur Anderson needs to identify the different input values assumed by each RBOC for each BSE study item. It would be useful for Arthur Andersen to identify the source of any facility usage assumption, as well; i.e., is it based upon actual RBOC data (like three-way calling holding times), forecasts, etc.

Facility usage units will involve traffic factors such as milliseconds of holding time, processor cycles, call CCS (hundred call seconds, average call duration) or BH CCS. They

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4. Indeed, the cost allocation rule in question was devised in order to create more equal conditions between costs incurred by RBOC enhanced services and services offered by competing providers. It makes sense to use the same time period in order to examine and compare calculations for BSEs that are supposed to benefit these same competing providers.

also involve, as the Bellcore presentation notes (p. 823.002), terminations, like various types of trunk terminations (e.g., at end office, at tandem switches, tie-trunk terminations, trunk and line card modules. Every such facility unit should be identified and subjected to sensitivity studies using the RBOC average number of units for a particular BSE versus the number input or assumed by a specific RBOC whose results are being subjected to he sensitivity analysis.

Finally, it is very important that the Arthur Andersen analysis be specified so that any redacted material should be carefully designed to fulfill the purposes of the FCC's non-disclosure order. As I understand it, the purposes of holding some of the SCIS-related confidential are (a) to preserve Bellcore's possible commercial interests in the software code and (b) to prevent the disclosure of data submitted by vendors of central office equipment. Lee Selwyn's memo of March 23, 1992 to you already has described why the redacted information made available to date cannot be used to provide an adequate analysis of the RBOCs' varying uses of SCIS and widely varying cost results. Additionally, that memo noted, and I hereby confirm based upon my personal knowledge, that the access to SCIS afforded by Bellcore in the context of the ONA tariff investigation is far more limited and restrictive than the access permitted in many state regulatory proceedings — where the state regulators have precisely the same objectives with respect to the protection of vendor data and software code as does the FCC.

Based upon my experience, it should be relatively easy to use the SCIS inputs, usage assumptions and relative outputs in order to perform the "benchmark" analyses that the ONA tariffs demand so clearly. Many facets of SCIS can be disclosed to parties who have been willing to sign the confidentiality agreement. Many of the illustrative "redacted" versions of the proposed Arthur Andersen output tables (generally page 2 of 2 of the Attachments) contain data that do not in any way compromise either the vendors' interests in the their price data or Bellcore legitimate interests in its intellectual property. This information simply cannot be "reverse engineered" so as to cause disclosure of such information. Many variables will have been blended together by the time Arthur Andersen will produce the types of output tables illustrated. Without detailed access to the types of limited data that *are* the subject of the confidentiality concerns, these data cannot be reverse engineered.[5] Arthur

5. I have utilized SCIS input and output data in several contexts, including comparing getting started cost estimates and assumptions, without ever looking at the underlying switch (continued...)

Andersen's March 30, 1992 letter transmitting the work plan to Bellcore, attached to the May 13 meeting material, does not appear to properly distinguish the types of data subject to redaction from the types of averaged, higher level cost *results* that should be disclosed under current practices.[6]

In some fifteen years of exposure to various versions and reiterations of the Switching Cost Information System I have found SCIS data to be quite helpful in analyzing LEC pricing practices. It may still prove useful in the FCC's context, but only if the Arthur Andersen audit and the resulting work products are carefully designed to address all issues that may be affected by SCIS and to produce meaningful *public* analysis.

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5. (...continued)
vendors' data and I have never had the need to examine the internal workings of the software itself.

6. By way of comparison, the Bureau's *NARUC ARMIS* letter (Richard Firestone to Paul Rodgers, November 7, 1989, FOIA Control No. 89-191) seems to make it clear that data will not be withheld from interested parties merely because it would embarrass the carrier or lead to results, such as changes in filed tariffs, deemed undesirable by the carrier. The same logic seems to apply here, except that the ARMIS matters partly involve data concerning non-regulated services that are presumptively competitive whereas BSEs are certainly monopoly services.

Peter H. Jacoby
Senior Attorney

Room 3245F3
295 North Maple Avenue
Basking Ridge, NJ 07920
908 221-4243

May 21, 1992

James E. Farmer
Arthur Andersen & Co.
33 West Monroe Street
Chicago, Illinois 60603

Re: Investigation of ONA Tariffs
<u>CC Docket No. 92-91</u>

Dear Mr. Farmer:

Pursuant to the Common Carrier Bureau's May 14, 1992 letter to intervenors in this proceeding, AT&T submits the following requested revisions to the methodology for review of the SCIS and SCM models described at the May 13 briefing from your firm. AT&T believes that these revisions will materially improve the results of the review process. AT&T also sets forth its understanding of certain activities that are already involved in the review process, or which your firm undertook to include in the review, based on the discussions at the May 13 briefing session. If, contrary to AT&T's understanding, any of these activities has not already been made part of the review process as modified through the May 13 discussions, AT&T requests that the review process be revised to include these items as well.

A. <u>Requested Revisions to the Review Process</u>

1. Arthur Andersen should review the criteria used in each company to select the offices chosen to create the model offices. If the BOCs used a sample of offices, Arthur Andersen should document the method for selecting these offices. Arthur Andersen should comment on the statistical validity of these selections.

2. Arthur Andersen should document the jurisdictional characteristics of the traffic data used in the studies that underlie the TRP data filed with the November 1, 1991 interstate ONA tariffs.

3. Arthur Andersen should identify and review the results of any audit, attestation, procedural review or similar study performed on SCIS and/or SCM that has been filed in any intrastate proceeding. Its redacted report should identify the person or company that performed each such study, the jurisdiction in which the study was filed and the title and/or docket number of the proceeding, and describe all relevant findings in those studies concerning these models.

4. Arthur Andersen should review any testimony filed in state proceedings regarding the reasonableness or reliability of the use of SCIS and/or SCM to provide investment and/or cost support. Arthur Andersen, as part of its redacted report, should list all of the proceedings and identify the testimony reviewed, and describe all factual information therein relevant to the findings in its current review.

B. <u>Understandings as to the Current Scope of Review</u>

1. AT&T understands that Arthur Andersen is currently performing tests, as part of its review on SCIS and SCM, that will measure the degree of closure in the models. (AT&T defines closure as the sum of all of the investment primitives plus all of the switch feature investments being equal to the total capital investment of each switch type, in both the "average" and "marginal" processing modes.) AT&T therefore expects that the degree to which the model misses the closure point will be documented by company and switch type in the redacted version of the report.

2. AT&T understands that Arthur Andersen is currently performing "sensitivity" analyses on all input variables in each switch type and company. AT&T expects that this information will be provided in the redacted report to the extent that switch vendor specific price discounts are masked to protect competitively sensitive information.

3. AT&T understands that Arthur Andersen will test the models, as part of its sensitivity analyses, for the variability caused by vendor list price changes and discounts. These tests will be performed by using a randomly selected vendor discount and then incrementally changing the discount value and measuring the change between these two points. This information will be provided in the redacted report because the discounts used will be a random number.

4. AT&T understands that Arthur Andersen will, as part of its sensitivity analysis, include an analysis based upon the most current version of the models and compare the results to those produced by the model version used for the November 1991 interstate ONA tariffs. Current model as of filing.

5. AT&T understands that Arthur Andersen will provide the results of its benchmark sensitivity analyses by interstate tariff entity for each BSE studied.

6. AT&T understands that Arthur Andersen will document the procedures used by the BOCs to aggregate their SCIS/SCM unit investments up to the interstate tariff entity using the TRP format.

7. AT&T understands that Arthur Andersen plans to further disaggregate each category on its "pie" chart of the causes of variability, so as to display the underlying causes in each category. For example, the sample chart displayed one category titled "Cost Methods." AT&T's understanding is that Arthur Andersen is planning to detail the contributing elements in this category on a separate pie chart.

Very truly yours,

Peter H. Jacoby

cc: Office of the Secretary, FCC
Chief, Tariff Division, Common Carrier Bureau

APPENDIX 17

SYMPOSIUM ON MARGINAL COST TECHNIQUES

ARTHUR ANDERSEN

ARTHUR ANDERSEN & Co. SC

SYMPOSIUM ON MARGINAL COST TECHNIQUES

A SYMPOSIUM ON MARGINAL COST TECHNIQUES FOR TELEPHONE SERVICES

CONDUCTED BY THE NATIONAL REGULATORY RESEARCH INSTITUTE

PAPER ON

BELLCORE'S SWITCHING COST INFORMATION SYSTEM (SCIS) COST MODEL

A PRACTICAL APPROACH TO A COMPLEX PROBLEM

SUBMITTED JUNE 20, 1990

BY

VIKTOR SCHMID-BIELENBERG

BELLCORE

DIRECTOR - SWITCHING AND NETWORK COST ANALYSIS

CONTENTS:

I. A description of Bellcore's Switching Cost Information System (SCIS) and how switching equipment engineering and provisioning rules (any technology or vendor) are transformed into mutually independent, physically significant investment functions.

II. An examination of various theoretical approaches consistent with economic theory that mathematically represent the "marginal" cost of new demand, and how they converge to what is commonly known as a "capacity cost".

III. An explanation of how the capacity cost approach is applied to the major investment functions of SCIS, how non-linearities of capacity added or investment added are handled, how "dual" limited equipment items are treated, and how engineered fill factors are accounted for.

IV. Given the above, practical solutions for determining the marginal investment of "island" features and services (switch based), intelligent network features and services (SS7 based), and for point to point voice, data, operator services, etc.

I. DESCRIPTION OF BELLCORE'S SWITCHING COST INFORMATION SYSTEM (SCIS)

a) GENERAL

In the mid nineteen sixties, Stored Program Control Switching Systems were beginning to appear in the topology of the switching network. Their predecessors, electromechanical switching systems, were poor cousins to these new generation switches in terms of flexibility and functionality. The older technologies offered only basic switched services for which there was practically no competition. To be sure, the No. 5 Crossbar switch offered an early version of CENTREX, but was limited to basics like intercom calling, attendant service and call transfer.

It is no wonder, that in those days, cost studies involving local switching were mostly addressing total cost recovery for revenue requirement purposes. It was not necessary to find a "cost" that could be used in deciding whether the general public and the firm would be better off with the introduction (or discontinuance) of a service. As providers of natural monopoly services, the question was not whether to continue or introduce a service, but rather how to best balance the concept of universal service, an allowed rate of return, and the greatest public welfare.

The only attempt in creating some similarity to a cost causation model was in the form of a simple formula, that acknowledged the existence of three significant cost drivers in a switch: Lines, Usage, and Calls. Or, simply (formula 1):

Total investment of a switch = (1)

Investment per Line (non traffic sensitive) * Lines
\+ Investment per CCS * Orig & Term CCS (Usage)
\+ Investment per Call * Orig & Term Calls

For cost studies, in those days mostly network services related, this provided sufficient accuracy. At worst, it failed to differentiate costs between a seven or ten digit call. No one had any inkling of the need for Feature Groups A, B, C, D pre and non pre subscribed, ANI, etc. and their impact on costs.

Today, the picture has changed dramatically. Processor controlled switches (analog and particularly digital) offer an ever increasing number of monopoly and competitive features and services. At last count, including various vendors of switches, SCIS had algorithms for over 800 of them. Another dimension of complexity is added with network intelligence. All, basic voice, data, and vertical services that heretofore were "island" services, now can use special signals (SS7) to communicate with distant switches, for an array of new capabilities with accompanying investment drivers and functions.

a) GENERAL (CONTINUED)

One thing is clear, the simplistic approach of the three part formula described earlier, is no longer sufficient. A more complex model is needed that will expand the granularity of the previous equation (formula 1) to recognize additional investment drivers. At the same time, the large initial (getting started) investments, as well as the addition of "lumpy" increments of capacity and investment over time, needs to be addressed.

From a practical perspective, one needs to build a model that can actually be used in a business decision process, given the real world limitations of time, available data, and tractability. To be sure, it needs to have sufficient granularity (investment primitives or building blocks) to be useful in determining cost causation for all available features and services, both near term and long term; it needs to adhere to widely accepted economic principles; and its results need to be understood and applied properly.

It needs to incorporate a process that does not arbitrarily assign common investments and expenses, but rather concerns itself with "what are the avoidable investments and expenses", by knowing the marginal investments and expenses of offering a new service.

This has an inherent vulnerability to those enamored with the traditional "cost plus" approach to pricing, therefore, a special word of caution. A marginal cost, does indeed provide a cost floor, and, thus, becomes an element in a pricing decision. But, by no means, does it provide all the intelligence required to determine the ultimate "best" price level.

This paper does not attempt to define this "best" price level, nor does it claim that SCIS will. SCIS, in fact stops at the investment level, and, only together with local methodologies that convert marginal investments into costs, identifies the cost floor, after direct expenses have been added.

Also not explained in this paper, are SCIS options that will permit a time value levelized equal allocation of spare processor capacity to all services for total cost recovery. Some of these options are available to satisfy wide ranging regulatory needs and applications, but do not pertain to the marginal cost theme of this paper.

I. A DESCRIPTION OF BELLCORE'S SWITCHING COST INFORMATION SYSTEM (SCIS) (CONTINUED)

a) GENERAL (CONTINUED)

SCIS methodology uses a philosophy of practical simplicity, without sacrificing expected accuracy or violating economic theory. This, we believe, is indeed the desire of all involved in providing cost support (cost analysts), in requiring cost support (regulators), in using cost support for business decisions (marketing and pricing groups), and in reviewing and defending cost support (economists and witnesses).

One should briefly define the relationships between investments and the final cost of a service. An investment, as used in this paper, is the vendor's charge (forward looking) to the local exchange carrier (LEC) for providing equipment required to perform switching functions for various features and services. It optionally includes vendor engineering and installation. Each local exchange service provider needs to go through a lengthy process to convert this investment to an annual recurring cost. This final cost considers all direct contributing factors that are required to "carry" or pay for, to house, to maintain, and to administer the investment.

A typical process involves these steps. Investments are computed through SCIS by entering usage data, call data, lines, trunks, engineered utilization factors, local discounts, etc. for one or more switches. This results in the vendor's expected charge to the LEC to provide equipment on a building block (investment primitive) and/or total basis. Then by entering feature or service specific data into SCIS, these investment primitives are utilized to determine specific investments for that feature or service, either switch specific, or, through a multi switch weighted average.

At this point, the LEC needs to add local engineering, installation, and other capitalized items to generate a total "in plant" investment. This is the total capital investment for switch equipment which can be converted to its annual "carrying charge" components (depreciation,income tax,etc.). If the equipment requires more building/land, similar components are computed for building/land investments. Equipment and buildings/land need to be maintained and incur administrative and other direct expenses. These become an additive component to the recurring capital expense components discussed before. The sum of all of these annualized recurring costs represent the yearly recurring cost of the investment that was generated by SCIS at the beginning of this process. This paper will utilize the term investment, recognizing that the above relationship exists.

The next section will explain how SCIS converts provisioning rules and capacities of major switch components into investment functions. The process, although technology and vendor specific, is consistent for all, and uses the same principles.

b) **TRANSFORMATION OF ENGINEERING AND PROVISIONING RULES FOR SWITCHING EQUIPMENT (ANY TECHNOLOGY OR VENDOR) INTO MUTUALLY INDEPENDENT, PHYSICALLY SIGNIFICANT INVESTMENT FUNCTIONS.**

A problem that concerns itself only with the total investment of a switch, can be solved by composing an equation that recognizes the important investment drivers of a switch. Then by varying these drivers over an accepted range of sizes, one can do multiple regressions or regression hyperplanes that have multiple correlations or partial correlations. The result might be of a form (formula 2):

$$\text{Total investment} = \text{Constant} + F\ (X_1,\ X_2,\ \ldots\ldots\ X_n) \qquad (2)$$

where X_1 to X_n are the investment drivers of the function F.

or of the form (formula 3):

$$\begin{aligned}\text{Total investment} = \text{Constant} + {} & F_1\ (X_1,\ X_i,\ \ldots X_m)\ + \qquad (3)\\ & F_2\ (X_i,\ \ldots\ldots X_n)\ + \\ & \quad\vdots \\ & F_j\ (X_m,\ \ldots\ldots X_n)\end{aligned}$$

where F_1 to F_j are functions, all with one or more investment drivers, some shared by several investment functions. At the same time, some of the drivers may be functions of other drivers. This creates situations, where each individual function has no physical significance, but the sum of all functions, predicts the total investment extremely well.

This approach, and sometimes approaches used in econometric models, can indeed be valuable predictors of the total investment of the switch, given a set of investment drivers. The problem is, that often such approaches, although good predictors of total investment changes, are not good predictors of the investment change caused, if only one or a few of the drivers are varied. Or, for our purpose, the above would not be good predictors of the marginal investment required for introducing a single additional feature. The individual functions or terms of the equation have no physical significance, and therefore can only be applied together, and not individually.

b) TRANSFORMATION OF ENGINEERING AND PROVISIONING RULES FOR SWITCHING EQUIPMENT (ANY TECHNOLOGY OR VENDOR) INTO MUTUALLY INDEPENDENT, PHYSICALLY SIGNIFICANT INVESTMENT FUNCTIONS. (CONTINUED)

This is the primary reason, that SCIS uses a bottom up engineering approach. It is important to know, what investments are caused by introducing a specific feature or service, and, how different aspects of that feature cause investment differences. For example, one might want to know, if the number of digits dialed varies in the activation code, what is its marginal investment.

Therefore, the SCIS approach uses multiple, single independent, but physically significant investment functions, that, when one or more of the drivers are applied will predict individually, and collectively, their effect on the individual and total investment. The equation for this is in the form of (formula 4):

$$\text{Total investment} = \text{Const} + F_1(X_1) + F_2(X_2) + \ldots + F_n(X_n) \quad (4)$$

where X_1 to X_n are independent investment drivers, and F_1 to F_n are each significant investment functions that depend on their driver and their investment driver only. An exception occurs, when an equipment unit has a dual capacity and ,therefore, two simultaneous investment drivers (See III.c.). Otherwise, no dependencies exist between investment functions. This is exactly what one needs in identifying marginal investments. Regardless of what changes, how it changes, and what unrelated event happens simultaneously, the marginal investment can be computed. This independence of terms is often lacking in top down models or models using regression hyperplanes (multiple correlations).

The process for building the SCIS approach is lengthy for the SCIS modelers at Bellcore. But it is manageable, if a consistent, systematic method is used. The advantage, once modeled, is that the SCIS system from a LEC cost analyst's perspective, is practical and relatively simple to use, although it has over 800 different features and services built into it. This is not to minimize the cost analyst's effort to gather local data for SCIS inputs and the effort to convert SCIS investments into costs as described earlier.

SCIS investment functions need to be created for each technology (analog and digital), vendor (A.T.& T., NTI, Siemens, Stromberg Carlson, Ericsson, etc.), type of switching node (host, remote type, tandem, ISDN and non-ISDN, SP/SSP, etc.) and major cost driver (identified later). To maintain its status as a forward looking cost model, SCIS is reviewed and updated periodically to reflect new equipment, new engineering rules, new vendor list prices, new capacities, etc.

b) TRANSFORMATION OF ENGINEERING AND PROVISIONING RULES FOR SWITCHING EQUIPMENT (ANY TECHNOLOGY OR VENDOR) INTO MUTUALLY INDEPENDENT, PHYSICALLY SIGNIFICANT INVESTMENT FUNCTIONS. (CONTINUED)

To actually derive the investment functions, one must obtain vendor provisioning rules, capacity tables, equipment interdependences, list prices, and other information proprietary to the vendor. This aspect, together with the intellectual property nature of the model development effort, often requires that SCIS information be kept confidential and proprietary.

The next step is to look at the functional aspects of a switch, and to determine what investment drivers present the smallest set required for determining physically significant investment building blocks or primitives of all features and services of a switch. Please refer to Figure 1.

These investment primitives and their drivers have been identified as follows:

1) A constant - The Getting Started Investment (Any equipment that needs to be purchased, independent of initial switch size or traffic served whenever a switch runs out of overall capacity of its most limiting resource). This is generally the main processor complex and any other equipment required when a whole new switch is put in to provide more capacity of that most limiting resource (ie. CPU time, milliseconds).

2) Milliseconds (Equivalent Calls or Half Calls) - This investment driver as well as physical terminations determine the Distributed Processor and related equipment investment.

3) Number of Lines (including administrative spare), by type (i.e. analog ,integrated digital loop carrier, ISDN) - The number of lines determine the investment for equipment dedicated on a one for one basis regardless of usage or calls (this is also called non traffic sensitive).

4) Originating and Terminating (O&T) CCS per Line, by type - This determines what type concentration is required on the line side, and the total line side CCS (O&T) determines the total line side CCS investment.

5) Outgoing and Incoming (O&I) CCS - This determines the total trunk side CCS investment.

6) Originating Calls - This determines the total investment for processing originating calls (any equipment used in the dialing of call - normalized to 7-digits).

b) TRANSFORMATION OF ENGINEERING AND PROVISIONING RULES OF SWITCHING EQUIPMENT (ANY TECHNOLOGY OR VENDOR) INTO MUTUALLY INDEPENDENT, PHYSICALLY SIGNIFICANT INVESTMENT FUNCTIONS. (CONTINUED)

7) Terminating Calls - This determines the total investment for processing terminating calls (any equipment used in the ringing of a line).

8) Outgoing Calls - This determines the total investment for processing of outgoing calls that use inband signaling equipment for notifying a distant office and transmitting digits.

9) Incoming Calls - This determines the total investment for receiving of incoming calls that use inband signaling equipment for acknowledging receipt of a call from a distant office.

Special Hardware is recognized through additional functions:

10) Touch Tone (DTMF) Calls - This determines total additional investment for DTMF equipment.

11) Feature Usage or CCS - This determines investment required for special hardware items such as 3-port circuits, 6-port circuits, announcement channels, etc.
Other hardware is provided on a one for one basis, such as make busy equipment for make busy keys, sensing equipment, etc.

12) Memory Bytes - This determines investment required for memory by type.

13) Signaling Octets - This determines the investment required for equipment used for out of band (SS7) signaling at the switch (SP/SSP).

14) ISDN Access Packets by type (per second) - This determines the investment required for ISDN access packets (non dedicated equipment, otherwise it is a function of lines).

15) Data packets by type (per second)- This determines the investment required at the switch to access the packet switched network or another ISDN colocated line for packet traffic.

b) TRANSFORMATION OF ENGINEERING AND PROVISIONING RULES FOR SWITCHING EQUIPMENT (ANY TECHNOLOGY OR VENDOR) INTO MUTUALLY INDEPENDENT, PHYSICALLY SIGNIFICANT INVESTMENT FUNCTIONS. (CONTINUED)

The above drivers are the smallest set required to capture all the investment functions for all vendors. Some of the investment functions may actually be zero, where a vendor's architecture or the available capacity does not require the purchase of any additional equipment to perform one of the described tasks. (i.e. varying a particular driver does not result in the purchase of additional equipment, either short term or long term).

For each switch type, each investment driver is examined to see what equipment components are a function of that driver. One cannot stop with the first component, but must continue examining all engineering rules until no other component as a function of that driver is found in a potentially long chain.

A very simple example is the provisioning of the non traffic sensitive part of the line. Each line may require a fraction of the investment of that main frame based on its line capacity, but it also has to be cabled out to a line peripheral, it also has to be protected for current surges, and that protection is mounted on some other piece of equipment, it also needs a test kit for so many lines, and on and on. Only when every engineering rule has been examined to see if anything in a long chain of events is still a function of the number of lines can one go on to the next investment driver.

By examining each of these drivers for different size switches, one can determine if the particular function is linear or non linear. Sometimes Poisson effects come into play as efficiencies of scale are achieved. This is particularly true in analog and electromechanical switches. However, it has been observed, that with very few minor exceptions, digital switch investment functions are linear.

Once this bottom up process for building physically significant investment functions (also called partitioning) has been completed by Bellcore, a specific total investment function can be created. This is in the form of formula 4 & 5, also referred to as the Model Office Equation (MOE). Please look at Figure 2.

I. A DESCRIPTION OF BELLCORE'S SWITCHING COST INFORMATION SYSTEM (SCIS) (CONTINUED)

b) TRANSFORMATION OF ENGINEERING AND PROVISIONING RULES FOR SWITCHING EQUIPMENT (ANY TECHNOLOGY OR VENDOR) INTO MUTUALLY INDEPENDENT, PHYSICALLY SIGNIFICANT INVESTMENT FUNCTIONS. (CONTINUED)

The model office equation is of the form (formula 5):

Total investment of any switch = (5)

Getting Started Investment
+ Distributed Processor Investment per millisec * milliseconds
+ Minimum Investment per Line * Lines (by line type)
+ Investment per O&T CCS * O&T CCS (by conc. ratio & line type)
+ Investment per O&I CCS * O&I CCS (by trunk type)
+ Investment per Originating Call * Originating Calls
+ Investment per Terminating Call * Terminating Calls
+ Investment per Outgoing Call * Outgoing Calls
+ Investment per Incoming Call * Incoming Calls

plus any special equipment:

+ Investment per DTMF Call * DTMF Calls
+ Investment per CCS for each Special Hardware(SH) Item * SH CCS
+ Investment per SH item * SH items
+ Investment per type of memory byte * memory bytes
+ Investment per signaling (SS7) octet * octets
+ Investment per ISDN access packet per second (pps) * pps (by type)
+ Investment per data pps * data pps (by type)

For rare cases, it is possible that one of the above call equations is of the form (non linear):

$A * \text{Calls}^{B}$, where A is a coefficient and B an exponent

If the model office equation has been developed correctly, it should now accurately predict the forward looking investment required to add any of the units expressed by its driver. Of course, it should be verifiable for any size switch, engineered and priced out by the vendor or by a mechanized tool provided by the vendor. The Bellcore model and the vendor tool reflect two different approaches, created by different teams. Therefore it is appropriate to consider the vendors tool used for actually ordering equipment as a base line for comparison to the Bellcore model. This is done over the complete range of capacities of the switch for the major part of the model office equation with an average error of less than three percent. The special hardware items required are directly linked back to vendor prices and provisioning rules, and are added vertically to the base investment.

II. AN EXAMINATION OF VARIOUS THEORETICAL APPROACHES CONSISTENT WITH ECONOMIC THEORY THAT MATHEMATICALLY REPRESENT THE "MARGINAL" INVESTMENT OF NEW DEMAND AND HOW THEY CONVERGE TO WHAT IS COMMONLY KNOWN AS A "CAPACITY COST".

a) GENERAL

For the purpose of discussion of the next part of this paper, Figure 3 is used to represent the most commonly experienced situation when dealing with Switching Equipment. As one can see, switching equipment is purchased, sometimes in rather sizable "lumps" of capacity. It may take several months or years, before this capacity is used up, and another similar unit needs to be added.

If one just added one unit of demand, and no immediate additional investment for the next unit is required, one could draw the conclusion that there is no marginal investment. This may be true for the short term, and, if appropriately applied, is the correct conclusion. This may also be true for the long term. If the remaining capacity of an equipment unit is so great, that it will never exhaust, then the introduction of a new service has no investment impact, and, therefore, the long run marginal investment is zero as well.

Although such an approach is appropriate for the instances cited above, it does not apply to the more common case in which the introduction of additional demand does cause the purchase or the advancement of a next equipment unit. Often, capacity is purchased in relatively large chunks, resulting in lumpy investment steps over time, which are advanced as additional demand is introduced. This advancement has a real economic cost, which is defined as the marginal cost (investment) in this paper. By not allowing the entry of the new demand, the advancement of the future investment steps is avoided.

Figure 4 shows a simple situation where a baseline case of demand triggered a new investment step every two years, as opposed to a base line case plus some added demand which advances the investment steps by one year, for ever. Although the impact in the first year is zero, the long term implication of adding demand is the advancement of future investments.

A numerical example is used in Figures 4A, 4B, 4C, and 4D to illustrate that the sum of the present values of the advanced investments (investment changes) divided by the sum of present value of the demand changes approaches the result obtained by simply taking the Investment of one unit and dividing it by its available capacity ("capacity cost"). Since that is not obvious nor totally intuitive, it can also be shown mathematically.

Proof I, was developed by Jay Lee and Viktor Schmid-Bielenberg, Bellcore, SCIS group, for the purpose of this paper, to show mathematically, why the numerical example of Figures 4 through 4D approached the "Capacity Cost" for a general case. (II. b.).

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a) GENERAL (CONTINUED)

Figure 5 is a more complex version of Figure 4; a situation that looks at added demand that does not have a constant relationship to the baseline (i.e. their slopes are not the same). A Memorandum by Ray Hayes, Bellcore in 1976 (formerly Bell Laboratories), case 36279-52, used a more general formula to prove that with practical assumptions, even this more complex form equals the "capacity cost".

Proof II, an adoption of the above for this paper, by Jay Lee and Viktor Schmid-Bielenberg, Bellcore, SCIS group, shows that this more complex case also equals the "capacity cost". (See II.b.)

Other variations occur, if the next investment step is dissimilar to the first one, the third one dissimilar to the second one, etc. (but the difference is not a simple function of inflation). This is the case with SS7 links and associated equipment. The investment for the first link pair is greater than the next, which is greater than the next, etc. Capacities added could vary as investments are added as well. (No Figure)

A modified "capacity cost" approach exists, even for this added dimension of complexity, but no proof will be presented for the sake of brevity.

Figure 6 (U S West Communications, SCM Group, San Diego Industry Forum on Telecommunications Costing in a Dynamic Environment, April 1989, Richard W. Foster and Robert M. Bowman) is similar to Figure 4 except that the demand increase materializes sometime after the last capacity addition at a redefined "To" (i.e. the first recovery period is shortened). As a consequence, the closer the new demand arrives to the exhaust of the present capacity, the greater its marginal cost burden, especially if one only includes the effect of the next advancement of investment.

This approach has the following short term marginal cost consequences: A low marginal cost, if demand arrives close to the last capacity installed; but a marginal cost spike or high marginal cost close to exhaust of the last capacity installed. The reason is that the time value of the change in investment added is divided by the time value of the change in demand. The denominator gets smaller the closer one gets to introducing the demand before the next investment is needed (Figure 6A). In a totally deregulated environment, with complete pricing flexibility, a low price encourages market entry, while spare capacity exists, and a high price discourages market entry near equipment exhaust. This tracks with the above marginal cost approach.

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a) GENERAL (CONTINUED)

If one were to "time value weight" the initially low marginal cost with the ever increasing marginal cost (until exhaust is reached) and then with the rapidly decaying marginal cost (after the next unit is installed), a steady state marginal cost is found, and, to almost no surprise, it also equals the capacity cost. Another way of saying this is, the same steady state cost (capacity cost) is reached if demand arrives randomly over the "To" period.

Proof III is an adoption of the above US West Communications formulation, by Jay Lee, Bellcore, SCIS group, and shows mathematically that the steady state marginal cost for this case also equals the "capacity cost".

All of the above, seemingly different situations, provide a uniform platform for using the "capacity cost" concept for an array of complex investment functions. Whenever equipment is provided over time, with lumpy investments and lumpy capacities added, this simplifying approach will not sacrifice accuracy nor economic validity.

Any alternative to this practical and tractable approach, would require a complexity that is not justified, and maybe impossible, given the limitations of data availability; of resources, human and other; and of turn around time to perform cost studies.

The benefits of this simplifying approach go way beyond the time saved by the cost analyst and the model developer. They also permit the use of verifiable cost support in regulatory proceedings, without having to rely on long term individual service forecasts, or other complex processes.

The next section, II.b., shows mathematical proofs justifying the use of what is commonly known as the capacity cost for an array of different situations. They are not all inclusive, for the sake of brevity, but they represent the most common real situations.

b) Mathematical Proofs

Common Definitions

For mathematical convenience discrete compounding $(1+i)$ is transformed into continuous compounding e^{δ} by letting

$$(1+i) = e^{\delta}. \tag{1}$$

By taking the natural log on both sides of the equation (1), one gets $\delta \ln e = \ln(1+i)$ or $\delta = \ln(1+i)$. One can always find a number δ such that $(1+i) = e^{\delta}$ and therefore e^{δ} can be used instead of $(1+i)$ without loss of validity and generality.

Also,

$$\frac{1}{(1+i)} = (1+i)^{-1} \tag{2}$$

$$= e^{-\delta}$$

This relationship will be utilized throughout the proofs.

Proof I.

Definitions:

To demonstrate the relationship between marginal investment and the capacity cost theory, the following notations are used (see Figures 3 and 4):

- q = capacity of machine or equipment unit (CAP)
- β = investment of machine or equipment unit (INV)
- ℓ_0 = initial demand at $t=0$
- d = an increase in demand
- δ = $\ln(1+i)$
- i = interest rate
- t_j = points in time at which an additional machine or equipment unit is added
- CC = capacity cost
- PVC = change in present value of investment
- PVD = change in present value of demand
- MC = marginal investment
- $LRMC$ = long run marginal investment

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b) Mathematical Proofs. Proof I. (Continued)

Hypothesis:

The capacity cost (CC) is defined to be:

$$CC = \frac{\text{investment of machine or next equipment unit}}{\text{capacity of machine or next equipment unit}} = \frac{\beta}{q} = \frac{INV}{CAP} \tag{1}$$

From the economic theory, the marginal investment (MC) is determined as:

$$MC = \frac{\Delta C}{\Delta Q} = \frac{PVC}{PVD} \; \therefore \; MC = CC \tag{2}$$

Detailed Equations:

If we assume that an increase in demand (d) is not greater than the remaining capacity of the machine or next equipment unit at $t=0$ $(0 \leq d \leq q-\ell_o)$, the first addition of a new machine or equipment unit will be made when

$\ell_o + d\,t_1 = q$ and

$$t_1 = \frac{q-\ell_o}{d} \tag{3}$$

Similarly, a second addition will be made when

$\ell_o + dt_2 = 2q$ and

$$t_2 = \frac{2q-\ell_o}{d} \tag{4}$$

In general,

$$t_j = \frac{jq-\ell_o}{d}, \quad j=1, 2, \cdots, \infty \tag{5}$$

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b) Mathematical Proofs, Proof I. (Continued)

Detailed Equations: (Continued)

The present value of investment (PVC) is determine as:

$$PVC = \sum_{j=1}^{\infty} \beta \cdot \frac{1}{(1+i)^{t_j}} \tag{6}$$

$$= \sum_{j=1}^{\infty} \beta \cdot e^{-\delta t_j}$$

$$= \beta \cdot \sum_{j=1}^{\infty} e^{-\delta\left(\frac{jq-\ell_0}{d}\right)}$$

$$= \beta\, e^{\frac{\delta \ell_0}{d}} \cdot \sum_{j=1}^{\infty} e^{\frac{-j\delta q}{d}}$$

$$= \beta\, e^{\frac{\delta \ell_0}{d}} \cdot \frac{e^{\frac{-\delta q}{d}}}{1-e^{\frac{-\delta q}{d}}} \tag{7}$$

$$PVD = \int_0^{\infty} d \cdot e^{-\delta t}\, dt = \frac{d}{\delta} \tag{8}$$

Then, the long run marginal investment ($LRMC$) is expressed as:

$$LRMC = \frac{PVC}{PVD} = \frac{\delta\beta \cdot e^{\frac{\delta \ell_0}{d}}}{d} \cdot \frac{e^{\frac{-\delta q}{d}}}{1-e^{\frac{-\delta q}{d}}} \tag{9}$$